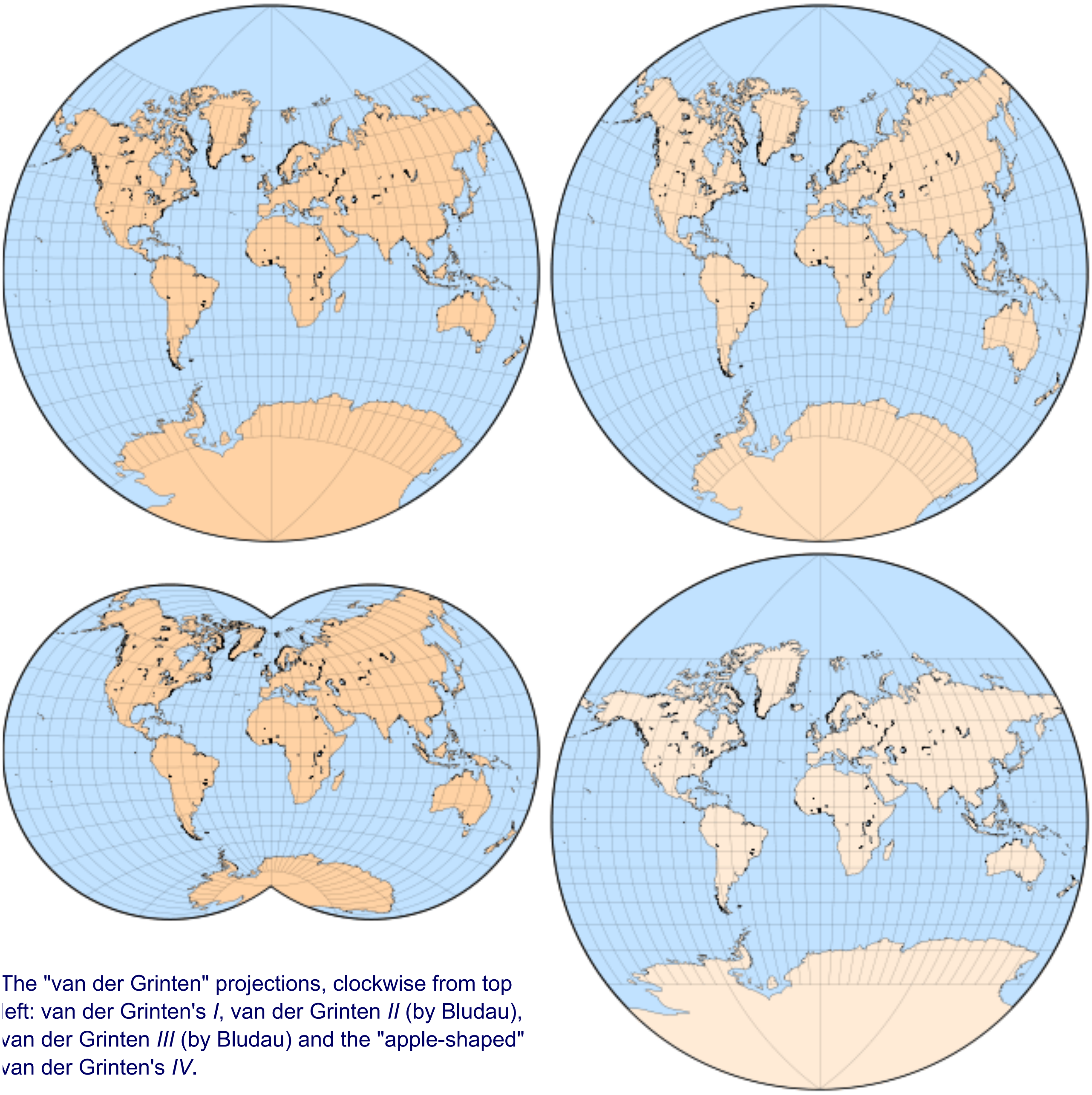
Map Projections

Other Interesting Projections



The "van der Grinten" projections, clockwise from top left: van der Grinten's *I*, van der Grinten *II* (by Bludau), van der Grinten *III* (by Bludau) and the "apple-shaped" van der Grinten's *IV*.

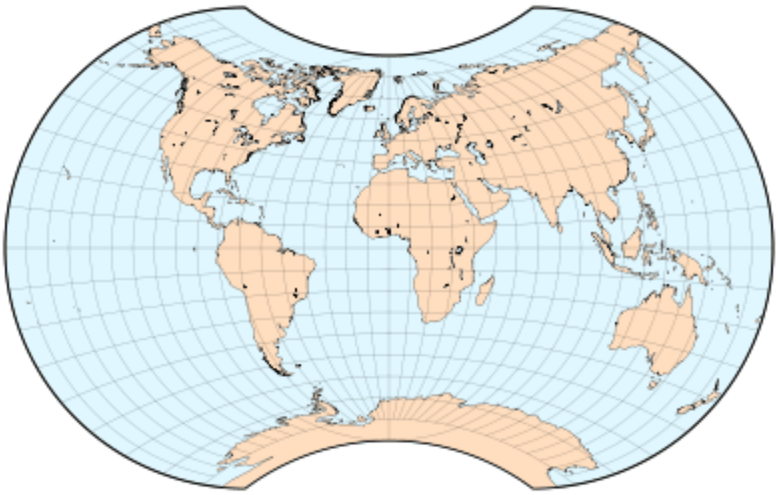
Countless projections were devised in centuries of map-making. Many designs cannot be readily classified in the main groups (azimuthal, cylindrical, pseudocylindrical, conic or pseudoconic), even though their design is similar or derived.
A large number of projections whose graticule lines are circles or derived conic curves with different radii and centers are called by some authors **polyconic** (not to be confused with the particular group of polyconic projections). This is a broad and artificial category comprising otherwise unrelated projections.

Projections by Van der Grinten

An American, Alphons J. van der Grinten published in 1904 and 1905 two projections, the first one devised as early as 1898. Both were designed for the equatorial aspect, with straight Equator and central meridian; all other parallels and meridians were circular arcs, with nonconcentric meridians regularly spaced along the Equator.

Alois Bludau proposed in 1912 two modifications to the first version; the four designs soon came to be collectively — and confusingly — called "van der Grinten" projections:

I. the first original projection, bounded by a circle
II. Bludau's modification of *I*, with parallels crossing meridians at right angles
III. Bludau's modification of *I*, with straight, horizontal parallels
IV. the second original projection, bounded by two identical circles with centers spaced 1.2 radii apart; the inner hemisphere is also circular



Maurer's "full-globular" map

Van der Grinten's proposals are examples of **conventional** designs, derived not from a perspective process but from an arbitrary geometric construction on the map plane. They are neither equal-area nor conformal (despite a superficial resemblance to projections by Lagrange, Eisenlohr and August), but intended to "look right", in the sense of conveying the notion of a round Earth (in this aspect, they resemble earlier globular projections) without departing too much from Mercator's familiar shapes.

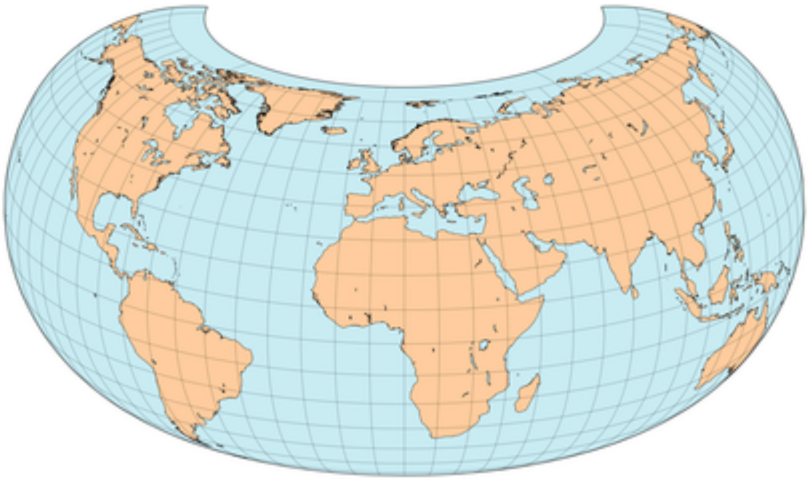
The best known of all four, van der Grinten's *I*, also known simply as *the* Grinten projection, was widely used, especially after its choice for reference world maps by the National Geographic Society from 1922 to 1988. Of the others, only the *III* variant saw limited use.
Although the poles can be included in the map, areal distortion is large at high latitudes, thus most van der Grinten maps are clipped near parallels 80°N and 80°S.

Globular Projections by Maurer

The ancient group of globular projections includes circular arcs for both meridians and parallels, and maps ordinarily limited to a single hemisphere.
H. Maurer presented in 1922 three conventional projections resembling globular features. The "full-globular" projection has meridians spaced like in van der Grinten's *IV* projection; parallels are equally spaced along the boundary meridians, and both the central meridian and the Equator have constant scale. Each boundary meridian spans half the limiting circle, thus the whole world is set resembling a double-edged ax.
His two other globular proposals are called "all-globular" and "apparent-globular".

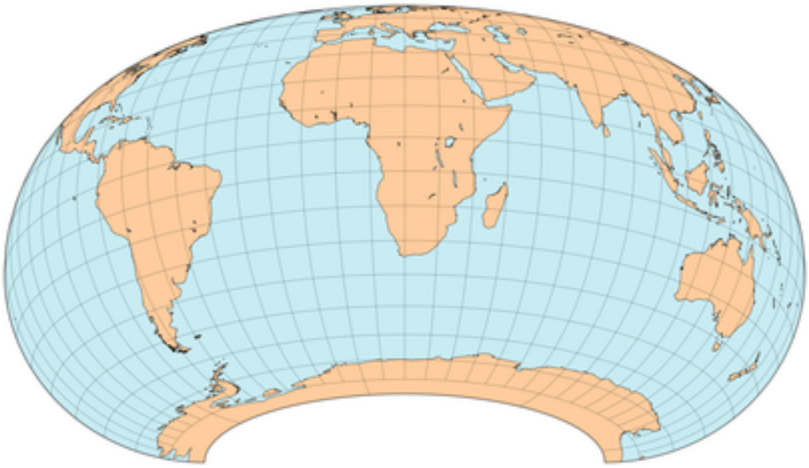
The Armadillo and other Orthoapsidal Projections

Beginning in 1943, the notable cartography teacher and author Erwin Raisz introduced a series of projections mapping the sphere onto intermediary curved surfaces. However, instead of "unrolled" like in cylindrical or conic maps, each surface is then projected orthographically onto the final plane. He coined the portmanteau "orthoapsidal", rooted on *apse*, from the Greek and Latin names for a vaulted recess.



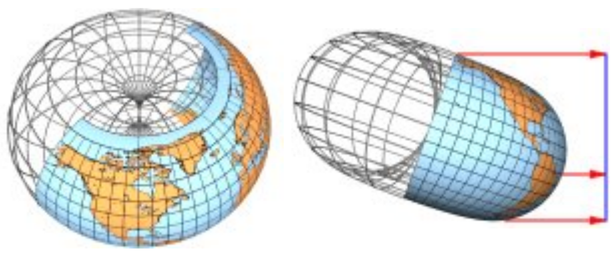
Reconstruction of the Armadillo projection as preferred by Raisz: tilted by 20°, central meridian 10°E, longitudes ranging from 170°E to 150°E with portions repeated to avoid splitting Alaska and Siberia.

In the most famous orthoapsidal projection, called "Armadillo" since it vaguely resembles the curling armored mammal, the sphere is mapped onto 1/4 of a degenerate torus with radii 1 and 1, which looks like a doughnut with a zero-sized hole. Parallels and meridians are equidistant circular arcs on the torus, but nonequidistant elliptical arcs in the final map.



Armadillo map emphasizing the southern hemisphere: tilt angle 20°, central meridian 20°E, no repeated portions.

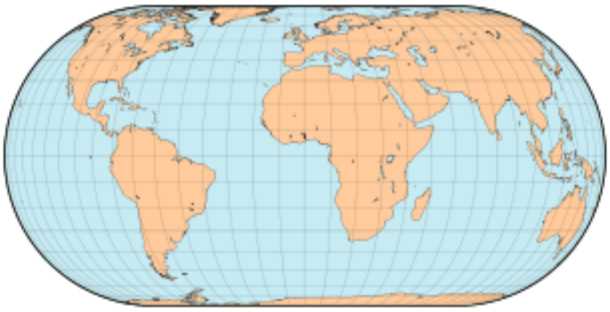
Schematic development of the Armadillo projection



After an equidistant mapping of the sphere to the region resembling half of a tire, the tilted region is orthographically projected into the blue plane.

In the conventional form of the Armadillo map, Raisz favored 10°E as the central meridian; the torus is then tilted by 20 degrees and orthographically flattened onto the projection plane. Parallels span more than 360°, leaving major landforms unsplit. Southern regions like New Zealand and Antarctica are hidden from view but can be presented as insets or extensions.

The simplest orthoapsidal designs suggested by Raisz had their construction outlined on one half of an oblate ellipsoid of revolution with



If the torus is not tilted, the result superficially resembles the third and fourth projections by Eckert.

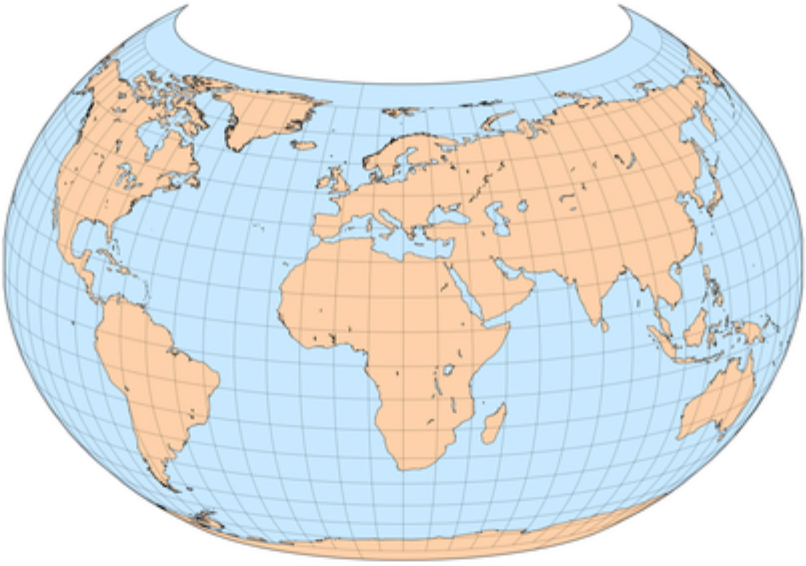
equatorial diameter twice the polar diameter — obviously this solid is completely unrelated to the reference ellipsoids adopted for large-scale conformal mapping with a datum. The first version was derived simply by squashing a sphere whose meridian spacing had been compressed to 50%, then tilting it by 20° and projecting it orthographically. Construction is straightforward and may be done geometrically, but the length of ellipsoidal meridians is about 54.2% too large compared with the ellipsoid Equator's, and scale is not constant along each meridian.

An orthoapsidal projection based on one half of an oblate ellipsoid of revolution with axes in proportion 1:2, therefore eccentricity 0.866, tilted by 20°. Poles are points, and the meridian scale is based on a squashed sphere's, as in Raisz's first proposal. Repeated portions with longitudes ranging from 150°E to 130°W, central meridian 10°E.

Raisz then recommended making the ellipsoidal meridian scale constant and identical to the Equators's; both poles become semicircular arcs. The appearance, superficially similar to the better-known Armadillo but with more of the southern hemisphere hidden, is generally improved but construction becomes much more difficult, requiring numerical approximation.
Raisz also mentioned changing meridian scale again in order to preserve areas, but he was probably referring to the ellipsoid instead of the final map.

Orthoapsidal projection according to Raisz's second proposal for the oblate ellipsoid with eccentricity 0.866 tilted by 20°: linear poles and constant meridian scale identical to the Equator's. Longitudes from 150°E to 130°W, central meridian 10°E.

Another surface employed by Raisz was one half of a tilted hyperboloid of revolution of two sheets; in this case, a North polar map was interrupted in four identical lobes, resembling Maurer's S231 projection and, different from other orthoapsidal designs, showing the whole world although considerably squashing the farther lobe. As drawn by Richard Edes Harrison, this projection was prominently featured on the cover of *Scientific American* 233(5); it is interrupted (at 60°E, 150°E, 120°W and 30°W) south of, apparently, 10°N. Harrison, known for his innovative and detailed maps, was quoted characterizing it as "the most elegant of all world maps".



Orthoapsidal projection on the oblate ellipsoid with eccentricity 0.661 and constant meridian scale. Longitudes from 160°E to 120°W, central meridian 20°E, tilt angle 15°.

Orthoapsidal projection on the prolate ellipsoid with eccentricity 0.6 and constant meridian scale, tilted by 20°. Longitudes from 175°E to 145°W, central meridian 15°E.

As originally designed, orthoapsidal maps are neither conformal nor equal-area; parallels and meridians do not necessarily hold properties (like equidistance) of the intermediary surface.

Raisz considered more exotic base shapes, like beans and scallop shells. He acknowledged the orthoapsidal principle would probably be more adequate for educational than thematic or scientific maps; on the other hand, he considered a viewer should unconsciously perceive orthoapsidal maps as three-dimensional representations, therefore recognizing the distortions as intrinsic to the projection process, not inherent to the represented regions.

Arden-Close's projection in hemispherical and whole-world maps

Conventional Eastern hemisphere, central meridian 70°E

Western hemisphere, central meridian 110°W

Extended to whole world

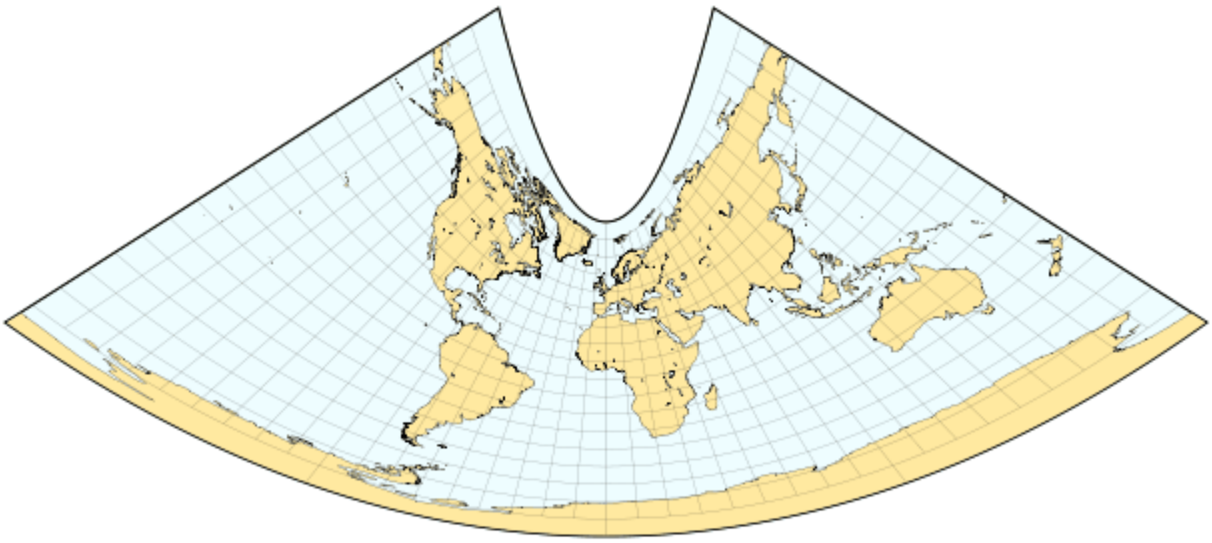
Whole world, transverse aspect

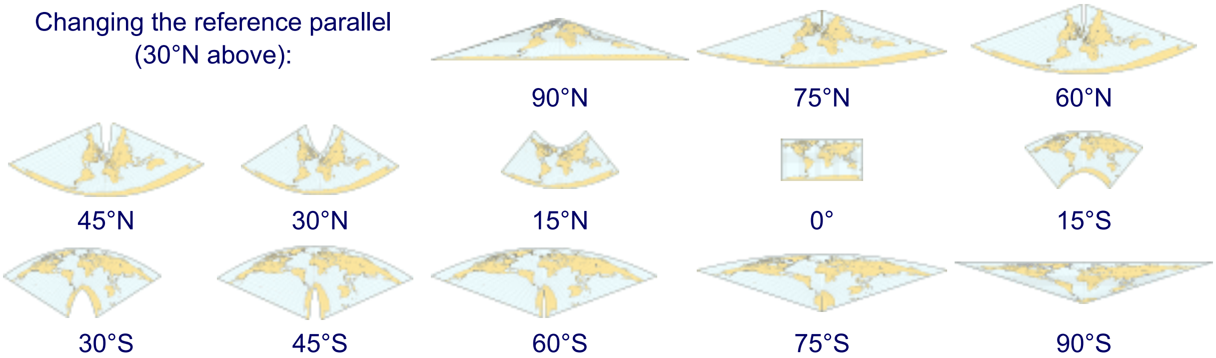
Averaged Projections by Arden-Close

Charles F. Arden-Close designed some map projections by averaging; his best-known proposal (1943) is a simple arithmetical mean of one hemisphere of an equatorial equal-area cylindrical map with its transverse aspect, the Equator in one map coinciding with the central meridian in the other. Shaped like a square with circular corners, the result is neither conformal nor equal-area.
Doubling coordinate values, his method can be easily extended in order to show the whole world in a single map.

Tobler's Projection for Local Maps

Sometimes, simplifying an existing projection may actually enhance its usefulness, or at least make it easier to use. That was Waldo Tobler's conclusion after looking for a projection suitable for efficiently presenting a small area such as the U.S.'s State of Michigan on a computer screen (1974). The requisites were fast computation, reasonable fidelity of shape and area, easily calculated distortion, simple parameterization, and exact and easily computed inverse equations, in order to quickly correlate screen and world coordinates.





Although Tobler never intended his projection for local maps to be used in world charts, it's an interesting exercise presenting how its distortion changes with the reference latitude.

After researching several conformal and equal-area approaches, Prof. Tobler decided for a previous projection by Tissot (1881), neither conformal nor equal-area, also designed for local maps. Tissot's projection is defined by a power series, but Tobler retained only the linear terms, which directly led to inverse equations. The projection is parameterized by a reference latitude, which also helped optimizing distortion. The trivial case, centered at the Equator, is identical to the Plate Carrée.

Gringorten's Projection

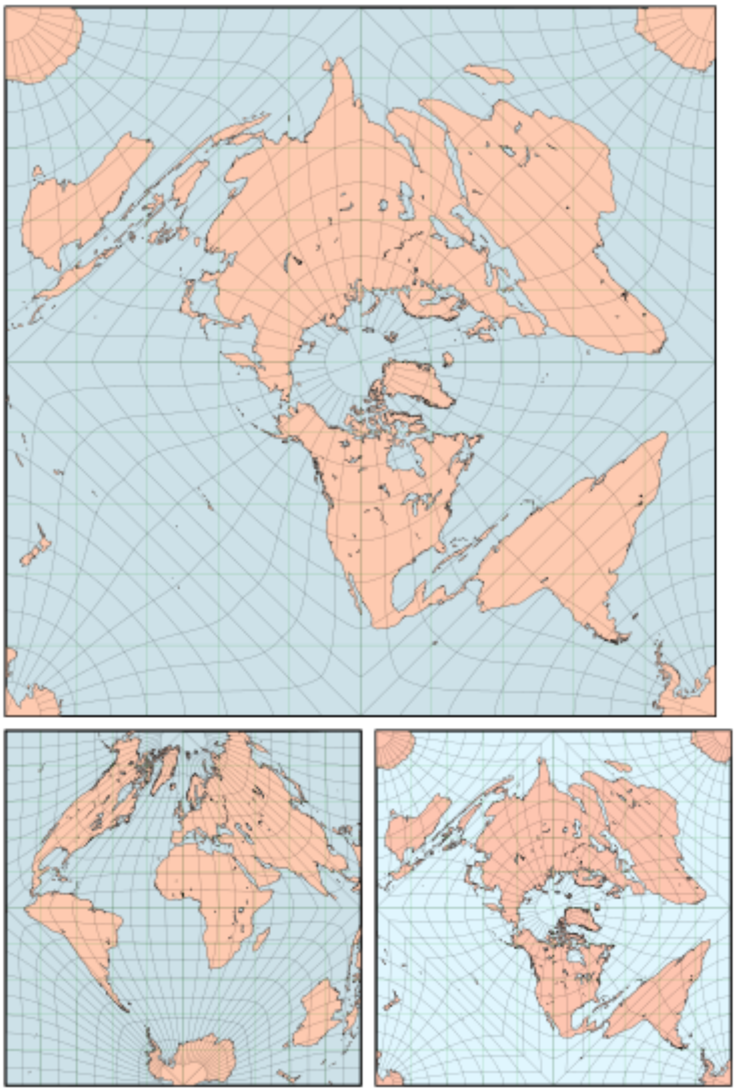
After looking for map projections suitable for global climate analysis, climatologist Irving I.Gringorten published in 1972 a very distinctive design, today nearly forgotten. He required a world map to be square, in order to efficiently use printed space in reports, articles and books; it should be equal-area so, for instance, the density and distribution of weather probes and stations can be correctly estimated at a glance; it should also avoid excessive shape distortion; and finally, should minimize continental interruption.

Details of Gringorten's projection were devised for a polar aspect of a spherical Earth, with a pole-centered hemisphere on an inner square, while the other hemisphere is split into four right triangles, its pole repeated across four corners. Except for interruptions, both hemispheres are fully symmetrical, an arrangement similar to Peirce's quincuncial map.
Each hemisphere comprises four triangular quadrants with a vertex on a pole, symmetrical around the central meridian. In each quadrant, parallels are elliptical

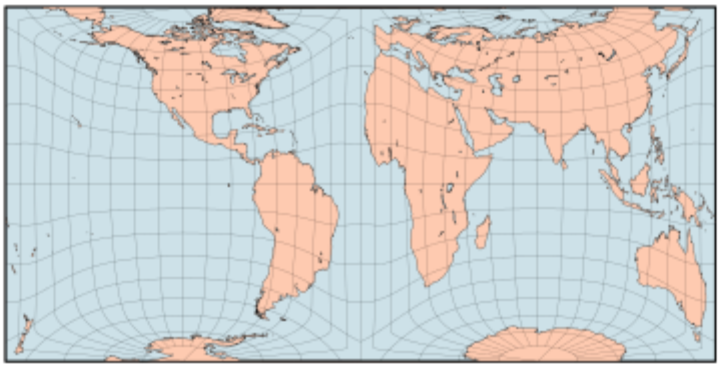
arcs, straight on the Equator and concave towards the pole. At quadrant boundaries, parallel arcs join with first order continuity. Given those constraints, there is more than one equal-area solution for meridian placement, unfortunately none satisfying another desirable attribute: meridians crossing the Equator at straight angles, thus unbroken across hemispheres.

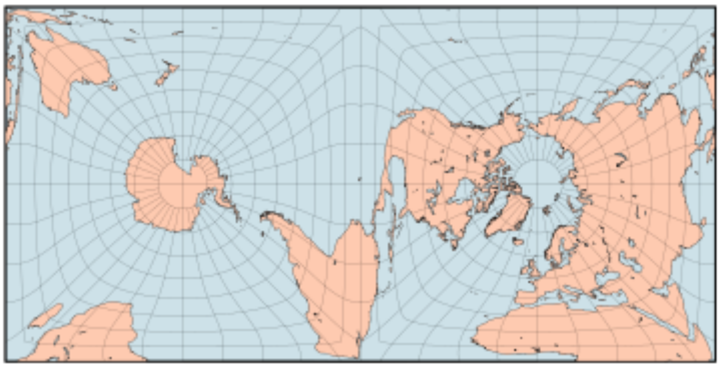
The mathematics demanded by Gringorten's projection is rather complex and must be calculated by numerical approximation. Gringorten's paper includes a table of computed coordinates and a north polar map, with southern lobes bounded by meridians 20°W, 70°E, 160°E and 110°W. The map is superimposed with a grid of 100 numbered cells, proposed as an aid for quickly locating points in addition to the ordinary latitude/longitude coordinate system. Antarctica is interrupted, but this inconvenience can be alleviated by an inset, by a second map with hemispheres swapped, or by moving three grid cells from corners to around the pole.

The author suggested other variations, like an oblique aspect and an alternative solution for meridian placement. Another immediate modification is rearranging the quadrant lay-out analogously to Peirce's and Guyou's maps: for instance, setting each hemisphere in a whole square.



Reconstruction of Gringorten's square map, with a superimposed 10x10 grid in normal (top) and transverse (above left) aspects. Above right, with alternative meridian placement. Below, quadrants rearranged giving each hemisphere an entire square.





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